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SHORTER COMMUNICATIONS

LAMINAR FREE CONVECTION FROM A HORIZONTAL CYLINDER WITH PRESCRIBED SURFACE HEAT FLUX

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NOMENCLATURE

a_1, a_2, wall surface heat flux parameters, equation (1);
Gr, Grashof number, $g\beta\,(T_w - T_\infty)R^3/\nu^2$;
Gr^*, modified Grashof number, $g\beta q_0 R^4/(k\nu^2)$;
k, thermal conductivity;
Nu, Nusselt number, $[q/(T_w - T_\infty)]\,(R/k)$;
Pr, Prandtl number, $\mu c_p/k$;
q, surface heat flux, $-(k\partial T/\partial x)_w$;
R, radius of cylinder;
T, temperature;
T_w, surface temperature;
T_∞, ambient temperature;
$\bar{x}$, dimensionless co-ordinate, (x/R_1) measured from the stagnation point along the surface;
β, expansion coefficient.

Subscripts

∞, at infinity;
iso, for isothermal wall;
0, at $\bar{x} = 0$;
q, for uniform surface heat flux;
w, at wall.

Superscripts

$-$, average.

Laminar free convection from horizontal cylinders has been studied in [1, 2] and [3] for isothermal and non-isothermal surfaces respectively. When the surface heat flux varies as equation [1], the free convection problem has been solved in [4].

$$\frac{q}{q_0} = 1 + a_1\bar{x}^2 + a_2\bar{x}^4. \tag{1}$$

The purpose of this note is to report the significant results of [4].

Surface temperature variation

For $Pr = 0{\cdot}7$ the surface temperature variation is given by

$$(T_w - T_\infty)\frac{k}{Rq_0}(Gr^*)^{1/5} = 2{\cdot}2142 + \bar{x}^2\,(1{\cdot}0831\,a_1 + 0{\cdot}04708) + \bar{x}^4\,(0{\cdot}88294\,a_2 - 0{\cdot}05405\,a_1^2 + 0{\cdot}02896\,a_1 + 0{\cdot}00157). \tag{2}$$

For $Pr = 1{\cdot}0$ the corresponding equation is

$$(T_w - T_\infty)\frac{k}{Rq_0}(Gr^*)^{1/5} = 1{\cdot}9963 + \bar{x}^2\,(0{\cdot}98737\,a_1 + 0{\cdot}04361) + \bar{x}^4\,(0{\cdot}80806\,a_2 - 0{\cdot}05020\,a_1^2 + 0{\cdot}02691\,a_1 + 0{\cdot}00150). \tag{3}$$

To ensure accurate results, the values of a_1, a_2, and $\bar{x}$ must be such that the second and third terms in equations (2) and (3) be relatively small in comparison with the first term.

Local Nusselt number

The local Nusselt number is obtained from:

$$\frac{Nu}{(Gr^*)^{1/5}} = \frac{q/q_0}{(T_w - T_\infty)(k/Rq_0)(Gr^*)^{1/5}} \tag{4}$$

where the numerator and denominator can be computed from equations (1) and (2) or (3) respectively.

It is interesting to compare the local Nusselt number for uniform surface heat transfer with that for uniform wall temperature. This comparison will be made for two separate conditions: (1) local values of $(T_w - T_\infty)$ are equal: (2) local values of heat flux are equal.

Case I. Consider a cylinder with uniform heat flux ($a_1 = a_2 = 0$). Equation (4) may be written as:

$$\frac{Nu}{Gr^{1/4}} = \left[\frac{Nu}{(Gr^*)^{1/5}}\right]^{5/4} = \left[\frac{1}{(T_w - T_\infty)(k/Rq_0)(Gr^*)^{1/5}}\right]^{5/4}. \tag{5}$$

Hence, the heat-transfer parameter $Nu/Gr^{1/4}$ as a function of $\bar{x}$ may be readily computed by use of equations (2) or (3) and (5). The value of Nu at $\bar{x}_i$ so computed is to be compared with that at x_i on another cylinder having a uniform temperature difference $(T_w - T_\infty)_{x_i}$. The results of this comparison are shown in curve I in Fig. 1. Since the comparison has been made for the same value of $(T_w - T_\infty)$ at x_i for the two cases, the ratio of Nusselt numbers as given by curve I in Fig. 1 is equal to the ratio of the local heat-transfer rates. From curve I of Fig. 1, it is seen that the local heat-transfer coefficient for the cylinder with constant heat flux is higher than that for

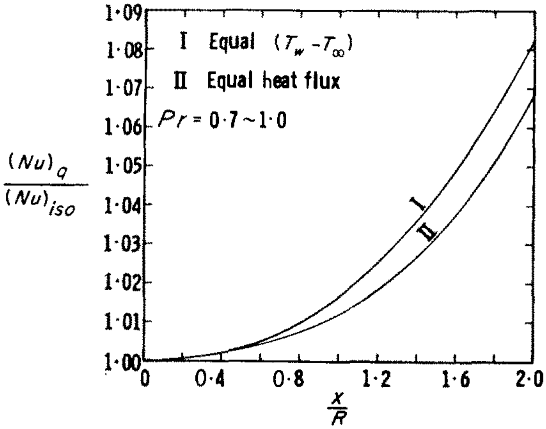

FIG. 1. Comparison of local Nusselt number.

the cylinder with constant wall temperature by about 8 per cent at $\bar{x} = 2$.

Case II. Consider a cylinder with a uniform surface temperature. It is known that the local heat-flux rate q varies with $\bar{x}$. At x_j there is a local heat-transfer rate q_{xj}. The ratio of local Nusselt number, for the same value of heat flux q_{x_j} may be expressed by

$$\frac{(Nu)_q}{(Nu)_{iso}} = \left(\frac{Nu}{Gr^{*1/5}}\right)_q \Big/ \left(\frac{Nu}{Gr^{1/4}}\right)_{iso}^{4/5}. \qquad (6)$$

The numerator may be computed from equation (4) while the denominator can be found from Reference 3. The results of calculation are shown in curve II of Fig. 1. It is evident from this curve that the Nusselt number ratio increases from 1 at the stagnation point to about 1·07 at $\bar{x} = 2$.

Average Nusselt number

The choice of a temperature difference in defining an average Nusselt number is quite arbitrary since there is no one temperature difference which is characteristic of the problem. In the case of a flat plate [5], an average Nusselt number has been evaluated based on two separate temperature differences: (1) an average temperature difference with the average taken over a range of values of $\bar{x}$. (2) A temperature difference between a midpoint on the surface and the free stream. These two separate temperature differences will be also used in computing an average Nusselt number for a horizontal cylinder:

(1) $\overline{Nu}$ based on an average temperature difference: The average Nusselt number may be written in terms of $\overline{T_w - T_\infty}$ as follows:

$$\frac{\overline{Nu}}{Gr^{1/4}} = \left(\frac{\overline{Nu}}{Gr^{*1/5}}\right)^{5/4} = \left[\overline{(T_w - T_\infty)}\,\frac{k}{Rq_0}\,Gr^{*1/5}\right]^{-5/4} \qquad (7)$$

The average temperature difference, $\overline{T_w - T_\infty}$, may be found by integrating equation (2) or (3) with respect to $\bar{x}$. The average Nusselt number as calculated from equation (7) for constant surface heat flux has been compared with that of a cylinder having a constant surface temperature. Such comparisons are shown in Table 1 for $\bar{x}_k = 2$.

Table 1. Comparison of average Nusselt number (Based on $T_w - T_\infty$ over $0 \leqslant \bar{x} \leqslant 2$)

Pr	$\left(\frac{\overline{Nu}}{Gr^{1/4}}\right)_q$	$\left(\frac{\overline{Nu}}{Gr^{1/4}}\right)_{iso}$	$\frac{(\overline{Nu})_q}{(\overline{Nu})_{iso}}$
0·7	0·356	0·348	1·02
1·0	0·405	0·396	1·02

From Table 1, it is seen that the average Nusselt number over $0 \leqslant \bar{x} \leqslant 2$ for constant surface heat flux is larger than that for constant surface temperature by only 2 per cent.

(2) $\overline{Nu}$ based on temperature difference at $(\bar{x}_k/2)$. The temperature difference between the surface and ambient at $(\bar{x}_k/2)$ may be computed from equations (2) or (3) by replacing $\bar{x}$ by $(\bar{x}_k/2)$. The resulting temperature difference may be substituted into equation (7) to evaluate $\overline{Nu}$. Again, a comparison in average Nusselt number has been made in Table 2 between a cylinder having a uniform surface heat flux and a cylinder having an isothermal surface. Table 2 indicates that the average Nusselt number (based on the temperature difference at $\bar{x} = 1$ when $\bar{x}_k = 2$) for uniform surface heat flux is larger than that for constant surface temperature by 4 per cent.

Table 2. Comparison of average Nusselt number (Based on $(T_w - T_\infty)$ at $\bar{x} = 1$)

Pr	$\left(\frac{Nu}{Gr^{1/4}}\right)_q$	$\left(\frac{Nu}{Gr^{1/4}}\right)_{iso}$	$\frac{(Nu)_q}{(Nu)_{iso}}$
0·7	0·360	0·346	1·04
1·0	0·410	0·394	1·04

Hence, the average Nusselt number for constant surface heat flux is not significantly different from that of an isothermal wall.

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SOME GENERALIZATIONS OF THE STABILITY OF LIQUID–GAS–VAPOR SYSTEMS*

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NOMENCLATURE

- c_p, specific heat at constant pressure;
- G, gas-content parameter defined in equation (5);
- g, specific Gibbs function;
- h, specific enthalpy;
- m, mass;
- p, pressure;
- p_{amb}, ambient pressure;
- p_v, vapor pressure evaluated at T_0;
- R, equilibrium bubble radius;
- R_{in}, stable equilibrium gas–vapor bubble radius for a given p_{amb};
- R_{ms}, maximum stable equilibrium gas–vapor bubble radius for a given G;
- R_0, unstable equilibrium radius of a vapor bubble;
- R_u, unstable equilibrium gas–vapor bubble radius for a given P_{amb};
- $\mathscr{R}$ gas constant on a unit mass basis;
- s, specific entropy;
- T, temperature ($\simeq T_{sat}$ if unspecified);
- T_{sat}, saturation temperature at p_{amb};
- V, volume;
- α, concentration of dissolved gas in a liquid;
- β, Henry's Law constant;
- Δa, thermodynamic availability above a given dead state;
- ΔG, potential barrier to nucleation;
- $\Delta\tau$, liquid superheat $[= (T_0 - T_{sat})]$;
- ρ, density;
- σ, surface tension between a liquid and its vapor.

General subscripts

- a, denoting permanent gas;
- f, denoting saturated liquid;
- g, denoting saturated vapor;
- l, denoting superheated liquid;
- o, denoting the locally superheated condition.

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INTRODUCTION

THE literatures of cavitation and of boiling have produced many worthwhile analyses [e.g. 1–6] on aspects of the stability of superheated and supersaturated liquid–gas–vapor systems. This note extends certain of this material to provide general equations and curves describing the limits of stability of such systems.

Bubbles grow spontaneously in supersaturated liquids because of mass diffusion into the liquid–gas interface, and in superheated liquids because of heat diffusion into the liquid–vapor interface with evaporation at the interface. In either case, a knowledge of the conditions on bubble stability with respect to growth or collapse aids in predicting growth inception and fixing initial conditions on dynamical equations.

THE PHYSICAL CONDITIONS ON STABILITY

A superheated or supersaturated liquid is in a condition of metastable equilibrium and can be perturbed into a state of unstable equilibrium by adding an appropriate spherical gas–vapor bubble. A static balance on such a perturbation bubble requires that:

$$p_a - p_{amb} + p_v = (2\sigma/R) \tag{1}$$

When there is a constant mass of permanent ideal gas, m_a, in the bubble:

$$p_a = \frac{3m_a \mathscr{R} T}{4\pi R^3} \tag{2}$$

but when mass diffusion is important:

$$p_a = \alpha\,\beta \tag{3}$$

In the former case:

$$\frac{(p_{amb} - p_v)}{2\sigma} = \frac{G}{R^3} - \frac{1}{R} \tag{4}$$

where: The *gas-content parameter*, $G \equiv \dfrac{3m_a \mathscr{R} T}{8\pi\,\sigma}$ (5)

Figure 1 displays equation (4) for 21 values of G, in completely general form. Figure 1 is similar to a curve